HYPER VERSION

Received: *January 12, 1998,* Accepted: *February 12, 1998*

Duality of chiral $\mathcal{N} = 1$ supersymmetric gauge theories via branes

Karl Landsteiner and Esperanza Lopez

Institut für theoretische Physik, TU-Wien
Wiedner Hauptstraße 8-10, A-1040 Wien, Austria
E-mail: landstei@tph45.tuwien.ac.at, elopez@tph16.tuwien.ac.at

David A. Lowe

Department of Physics, Brown University
Providence, RI 02912, USA
E-mail: lowe@het.brown.edu

Abstract: Using a six-orientifold on top of a NS-fivebrane we construct a chiral $\mathcal{N} = 1$ supersymmetric gauge theory in four dimensions with gauge group $SU(N_c)$ and matter in the symmetric, antisymmetric and (anti)fundamental representations. Anomaly cancellation is fulfilled by the requirement of a smooth RR 7-form charge distribution and leads us to the introduction of 8 half D-sixbranes ending on the NS-fivebrane. We obtain the dual model from branes by a linking number argument. We check explicitly the 't Hooft anomaly matching conditions and the map between deformations in the original and the dual model.

Keywords: Branes in String Theory, Supersymmetry and Duality.

JHEP02(1998)007

Contents

1 Introduction

In the last year Dirichlet brane [1] techniques have been proven to be an extremely powerful tool for the study of non-perturbative phenomena in supersymmetric gauge theories. A collection of parallel D-p-branes would give gauge theories with 16 supercharges. In order to reduce the number of supersymmetries one can use the fact that D-branes can end on certain other branes [2]. In Type IIA string theory D-fourbranes are allowed to end on Neveu-Schwarz (NS)-fivebranes. Connecting parallel NS-fivebranes by a set of D-fourbranes leads then to an effectively four-dimensional $\mathcal{N}=2$ gauge theory [3]. Matter hypermultiplets transforming in the fundamental representation can be introduced by including D-sixbranes. When a D-sixbrane crosses one of the NS-fivebranes a D-fourbrane stretched between the NS-fivebrane and the D-sixbrane is created [4] and this leads to an alternative way of introducing hypermultiplets, i.e. one gets a hypermultiplet from strings stretching between fourbranes that end on different sides of a NS-fivebrane. Configurations with more than two NS-fivebranes lead to theories with product gauge groups and hypermultiplets transforming in the bifundamental representation of two neighboring gauge groups. Furthermore it has recently been shown that by putting an orientifold sixplane on top of a NS-fivebrane one gets hypermultiplets in the symmetric or antisymmetric representation of $SU(N_c)$ [5]. These brane configurations can be lifted to M-theory. The D-fourbranes are then M-fivebranes wrapped around the eleventh dimension. In this limit one deals with a single smooth M-fivebrane wrapped on a complex one dimensional curve. The complex curves are precisely the ones which parameterize the Coulomb-branch of $\mathcal{N}=2$ gauge theories.

By rotating the NS-fivebranes with respect to each other one can further reduce the amount of supersymmetry. Choosing in particular angles that correspond to $SU(2)$ rotations in a four-dimensional embedding space one can construct brane configurations with $\mathcal{N}=1$ supersymmetry in four dimensions [6]. It was shown in [7] that Seiberg's dualities [8] can be recovered by moving on the moduli space of the brane configuration. An important feature that is present for $\mathcal{N}=1$ but not for $\mathcal{N}=2$ is chirality. A mechanism for enhanced chiral symmetry in the brane configuration has been proposed in [9]. There it was suggested that when N_f D-sixbranes meet a NS-fivebrane such that they are divided in two pieces one gets an enhanced $SU(N_f)_R \times SU(N_f)_L$. This leads to the picture that strings stretching from half D-sixbranes ending on NS-fivebranes to the D-fourbranes give rise to chiral $\mathcal{N}=1$ multiplets. The fact that the Ramond-Ramond (RR) 7-form charge is not conserved for such a configuration is interpreted as the anomaly in the $\mathcal{N}=1$ gauge theory [10]. In order to generate anomaly free chiral models with a simple gauge group one needs matter in two index tensor representations[1]. Thus one would expect that by combining the methods of [10] with that of [5] one should be able to construct brane configurations realizing anomaly free $\mathcal{N}=1$ gauge theories. We show that this is indeed true.

2 Brane configurations

Let us begin by reviewing the basic brane configuration we will use to realize four-dimensional gauge theories with $\mathcal{N}=1$ supersymmetry. The system of branes includes three different NS-fivebranes. The central fivebrane (labeled B) extends in the $(x^0, x^1, x^2, x^3, x^8, x^9)$ directions, while the outer fivebranes (labeled A and C) are tilted at $SU(2)$ angles from the (x^8, x^9) plane towards the (x^4, x^5) plane. We also have a number of D-fourbranes stretching between the fivebranes extended along the $(x^0, x^1, x^2, x^3, x^6)$ directions. Finally we have three different types of D-sixbranes, which stretch parallel to the three different kinds of fivebranes, but are also extended in the x^7 direction. This configuration of branes gives rise to a gauge theory with $\mathcal{N}=1$ supersymmetry in the four dimensions (x^0, x^1, x^2, x^3), for arbitrary angles.

The gauge group and matter content are determined by the following rules.

1) n D-fourbranes stretching between two fivebranes will give rise to a massless vector multiplet with $SU(n)$ gauge symmetry. When the fivebranes are parallel there will be an additional chiral multiplet in the adjoint representation, however this multiplet becomes massive when the branes are at an angle.

2) A D-fourbrane between two D-sixbranes gives rise to two chiral multiplets.

3) When a D-fourbrane stretches between a D-sixbrane and a parallel NS-fivebrane, there is a single chiral multiplet. If the fivebrane is not parallel to the sixbrane, no massless matter appears.

[1]A different way of obtaining anomaly free chiral models from branes has been proposed in [11]. Here one puts the brane configuration in an orbifold background. The chiral matter is produced by the orbifold projection rather than by half D-sixbranes.

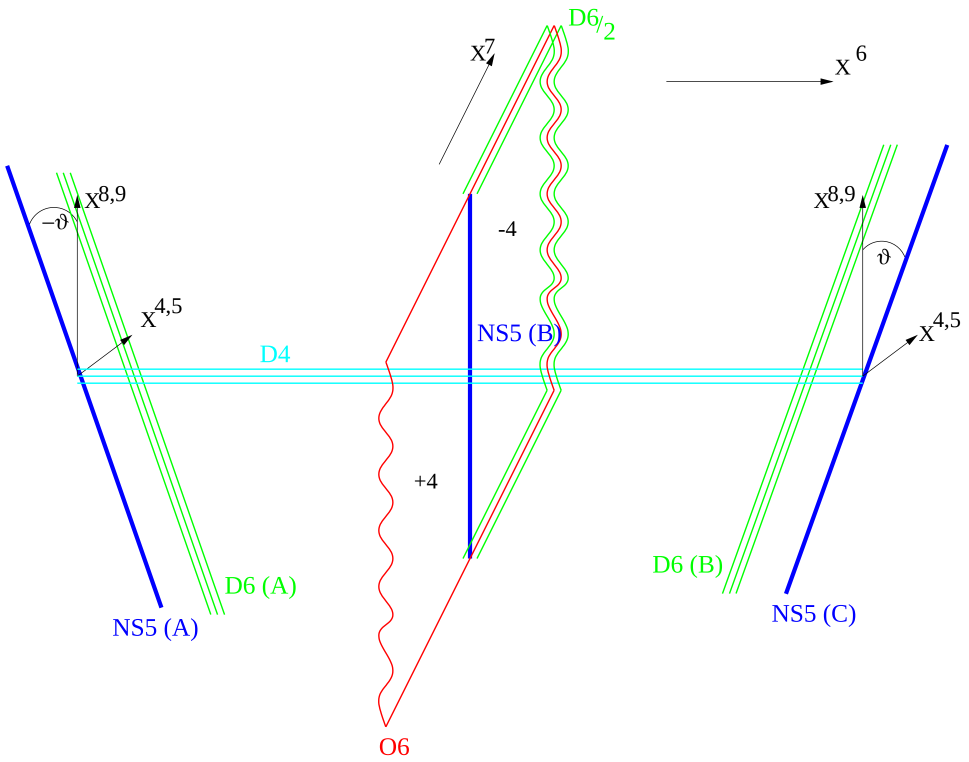

Figure 1: A configuration of three fivebranes connected by N_c parallel fourbranes and an orientifold sixplane. The outer fivebranes are rotated by an angle ϑ with respect to the middle one. The O6 is divided in half by the middle fivebrane in the x^7-direction and changes its character there. To compensate for the jump in the sixbrane charge of the orientifold we also have to put eight half D-sixbranes on top of the orientifold where it has charge -4. These sixbranes end on the middle fivebrane. There are also m_f additional sixbranes (and their mirror images) tilted in the same way as the fivebranes. They also extend along the x^7-direction (this is not shown in the figure).

4) D-fourbranes on one side of an NS-fivebrane give rise to a chiral multiplet in the fundamental and one in the anti-fundamental representation of the gauge group associated with fourbranes on the other side of the NS-fivebrane.

5) When a D-sixbrane meets a D-fourbrane in space a chiral multiplet in the fundamental and one in the anti-fundamental representation is generated. As argued by Brodie and Hanany [9], when the sixbranes touch a parallel fivebrane, chiral symmetry is restored, and the sixbranes may split.

To construct a gauge theory with truly chiral matter we will use one more element, an orientifold sixplane, which we will place on the central fivebrane, extending in the $(x^0, x^1, x^2, x^3, x^7, x^8, x^9)$ directions. The brane configuration is shown in fig. 1. The orientifold acts as a reflection taking $(x^4, x^5, x^6) \to (-x^4, -x^5, -x^6)$. From now on all branes will be arranged to respect this $\mathbb{Z}_2$ symmetry.

Orientifolds have been used in previous works [12, 13, 5, 14] to construct gauge theories with orthogonal and symplectic gauge groups using brane constructions. There are two possible choices of orientifold projection, one of which gives rise to orthogonal gauge groups on the D-fourbranes with a symplectic symmetry appearing on D-sixbranes. For the case of an orientifold sixplane, the sixplane carries RR charge equivalent to 4 sixbranes. It has been argued that this orientifold is equivalent to a D_4 singularity from the eleven-dimensional M-theory point of view [5, 15]. The other choice of projection gives rise to symplectic groups on the D-fourbranes, and orthogonal groups on the D-sixbranes. This orientifold sixplane carries -4 RR charge, and has been argued to be equivalent to the Atiyah-Hitchin space from the M-theory point of view [16, 17].

In [12], it was argued that when an orientifold fourplane traverses a NS-fivebrane the type of projection changes. In our case an orientifold sixplane crosses the NS-fivebrane. This is T-dual to the situation considered in [12]. Thus the orientifold projection must change when the sixplane crosses a NS-fivebrane. In the case at hand, this means the projection will change as one moves from positive x^7 to negative x^7. We also place 8 half D-sixbranes on the fivebrane to cancel the discontinuity in the RR charge. This will lead to a smooth brane configuration from the eleven-dimensional point of view, which will allow a geometric analysis of the $\mathcal{N} = 1$ duality in the spirit of [7].

The gauge group of this configuration may be determined following [5] to be $SU(N_c)$ where N_c is the number of D-fourbranes. Similar arguments imply the matter content is chiral with a chiral multiplet in the symmetric representation, a chiral multiplet in the antisymmetric representation, and $m_f + 8$ chiral multiplets in the fundamental, and m_f chiral multiplets in the anti-fundamental. The matter content is the same as one of the models considered in [18]. However in our situation the global symmetries and the superpotential are different. The two classes of models are related by deformations, as we will see later.

Let us consider the superpotential that arises from this brane configuration. We will argue it takes the following form

$$W = \frac{1}{\mu}\mathrm{Tr}\ (X\tilde{X})^2 + \hat{Q}\tilde{X}\hat{Q}\ , \tag{2.1}$$

where X is the antisymmetric field, $\tilde{X}$ is the symmetric field and $\hat{Q}$ are the 8 fundamental fields arising from the 8 half D-sixbranes. The $(X\tilde{X})^2$ term is seen to arise as in [6, 9]. In our case, the two outer fivebranes A and C are rotated by equal but opposite angles, with respect to B. When the angle is zero we have an additional massless chiral multiplet Φ in the adjoint representation. There is a coupling of the form $\tilde{X}\Phi X$, but no coupling of Φ to the fields Q, $\hat{Q}$ and $\tilde{Q}$. This is because the sixbranes are parallel to the fivebranes. For finite angle, this chiral multiplet gets a mass given by $\tan\vartheta$. Integrating out this field gives the $(X\tilde{X})^2$ term, with μ equal to the mass of the adjoint chiral multiplet.

The 8 half D-sixbranes give rise to eight chiral multiplets in the fundamental representation. Taking also the m_f chiral multiplets from the other D-sixbranes one would naively expect an $SU(m_f + 8)_L$ flavor symmetry. The flavor symmetry should appear as the gauge group on the sixbranes. This is however not what the brane configuration tells us. The 8 half D-sixbranes sit on top of an orientifold. Therefore the flavor symmetry is $SU(m_f)_L \times SO(8)_L$. The $\hat{Q}\tilde{X}\hat{Q}$ term in the superpotential gives us precisely this breaking of the flavor symmetry. The origin of the $\hat{Q}\tilde{X}\hat{Q}$ term can also be seen when one considers detaching the fourbranes from the central fivebrane, and then moving the fivebrane in the positive x^7 direction. The $\hat{Q}$ fermions clearly become massive under this deformation. The gauge group is broken to $SO(N_c)$ because now the fourbranes will cross the orientifold sixplane in the region which gives a projection to the orthogonal gauge group. The position of the fivebrane in the x^7 direction is a single real parameter, and plays the same role as the Fayet-Iliopoulos (FI) parameters that appear in [7]. While the $U(1)$ component of the gauge group is frozen, the FI parameters still appear in the D-term equations of the low-energy field theory

$$\nu\delta_a^b = Q_a^f(Q^\dagger)_f^b + \hat{Q}_a^i(\hat{Q}^\dagger)_i^b - (\tilde{Q}^\dagger)_a^{\dot{g}}\tilde{Q}_{\dot{g}}^b + 2X_{ac}(X^\dagger)^{cb} - 2(\tilde{X}^\dagger)_{ac}\tilde{X}^{cb} , \tag{2.2}$$

where ν is related to the x^7 displacement of the fivebrane, and we have also introduced the fundamental fields Q and the anti-fundamentals $\tilde{Q}$. We see then that taking ν to be negative (i.e. moving in the positive x^7 direction), and assuming the fundamental, anti-fundamental and antisymmetric squarks do not get a vev, $\tilde{X}$ gets a vev. In the simplest case where only the central fivebrane is moved, we expect this to correspond to a baryonic branch characterized by $(\tilde{X})^{N_c}$ receiving a vev. This vev generates a mass for the $\hat{Q}$'s provided the $\hat{Q}\tilde{X}\hat{Q}$ term appears in the superpotential. Another way to see that the $\hat{Q}\tilde{X}\hat{Q}$ appears in the superpotential is to consider moving the central fivebrane in the negative x^7 direction, which corresponds to an expectation value for the baryon $X^{N_c/2}$, for N_c even. The $\hat{Q}$ fermions will remain massless in this case, and the gauge group will be broken to $Sp(N_c)$ since now the fourbranes cross the orientifold plane on the side that gives the symplectic projection. The $\tilde{X}$ field becomes the adjoint of $Sp(N_c)$ with finite mass dependent on the angle ϑ. If we set $m_f = 0$ and rotate to $\vartheta = \pi/2$, $\tilde{X}$ will be massless and $\mathcal{N} = 2$ supersymmetry will be restored, which requires a coupling of the form $\hat{Q}\tilde{X}\hat{Q}$. We will postpone further discussion of these baryonic branches to the next section.

From the brane point of view, the transition to the magnetic dual theory is seen by moving the branes to a mirror image configuration, and then moving the sixbranes outside the fivebranes. This is shown in fig. 2. Since for our brane configuration we do not expect the orientifolding to introduce any extra singularities, we may use the linking number argument [4] to determine how many D-fourbranes we end up with after rearranging the fivebranes and sixbranes. We assume a fourbrane is created whenever a fivebrane crosses a sixbrane which is non-parallel [9]. In addition, we assume that when a fivebrane crosses the orientifold plane, 4 D-fourbranes are created (i.e. the orientifold plane is treated as set of 4 D-sixbranes as far as the linking number argument goes). The linking number of a fivebrane is given by $L_5 = \frac{1}{2}(n_{6L} - n_{6R}) + (n_{4R} - n_{4L})$, where n_{6L} is the number of non-parallel D-sixbranes to the left, etc. Similarly the linking number of a D-sixbrane is given by $L_6 = \frac{1}{2}(n_{5L} - n_{5R}) + (n_{4R} - n_{4L})$ where $n_{5L,R}$ denotes now the number of non-parallel fivebranes to the left or to the right of the D-sixbrane under consideration. These numbers are conserved under brane moves. If we focus on a single A sixbrane and the fivebrane A, their initial linking numbers are $L_6^A = -1$ and

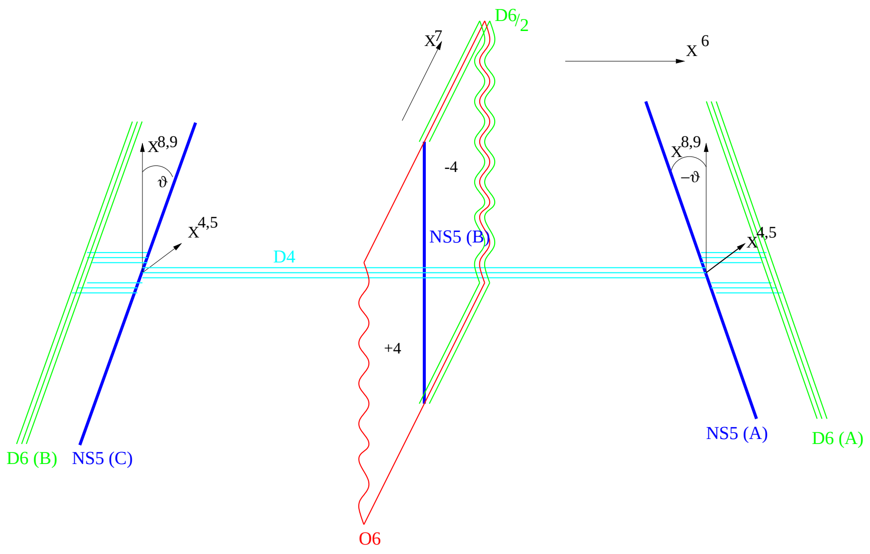

Figure 2: The magnetic brane configuration

$L_5^A = -\frac{1}{2}m_f - 2 + N_c$. In the final configuration, the linking number of the D-sixbrane is $L'^A_6 = 1 - n_4$, where n_4 is the number of D-fourbranes connecting an A-D-sixbrane with the fivebrane A. Conservation of the linking number gives $n_4 = 2$. The linking number of the fivebrane is $L'^A_5 = 2 + \frac{5}{2}m_f - \tilde{N}_c$. With this we find $\tilde{N}_c = 3m_f - N_c + 4$, and thus the dual gauge group is $SU(\tilde{N}_c)$ with chiral multiplets in the symmetric and antisymmetric representations, and m_f+8 fundamental chiral multiplets, and m_f anti-fundamental chiral multiplets. In addition, we have a number of gauge singlets corresponding to meson fields.

3 Duality of chiral models

3.1 Electric theory

The gauge group is $SU(N_c)$ with a symmetric and an antisymmetric chiral multiplet together with m_f+8 fundamental and m_f anti-fundamental chiral multiplets. The anomaly-free global symmetry group is $SU(m_f)_L \times SU(m_f)_R \times SO(8)_L \times U(1)_B \times U(1)_X \times U(1)_R$. The fields live in the representations listed in table 1.

	$SU(N_c)$	$SU(m_f)_L$	$SU(m_f)_R$	$SO(8)_L$	$U(1)_B$	$U(1)_X$	$U(1)_R$
$\hat{Q}$	$\mathbf{N_c}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8_v}$	$\frac{1}{N_c}$	$\frac{1}{2}$	$\frac{3}{4}$
Q	$\mathbf{N_c}$	$\mathbf{m_f}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{1}{N_c}$	0	$1-\frac{N_c-2}{2m_f}$
$\tilde{Q}$	$\overline{\mathbf{N_c}}$	$\mathbf{1}$	$\mathbf{m_f}$	$\mathbf{1}$	$-\frac{1}{N_c}$	0	$1-\frac{N_c-2}{2m_f}$
X	$\mathbf{asym}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{N_c}$	1	$\frac{1}{2}$
$\tilde{X}$	$\overline{\mathbf{sym}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{N_c}$	-1	$\frac{1}{2}$

Table 1: Electric theory: representation of fields.

Setting μ to 1 for convenience, the superpotential takes the form

$$W = \text{Tr }(X\tilde{X})^2 + \hat{Q}\tilde{X}\hat{Q}\ , \tag{3.1}$$

and truncates the chiral ring via the relations

$$\tilde{X}(X\tilde{X}) = 0, \quad 2(X\tilde{X}X)^b_a + \hat{Q}_a\hat{Q}^b = 0, \quad \tilde{X}\hat{Q} = 0\ . \tag{3.2}$$

The chiral mesons are $M_0^{f\dot{g}} = Q^f\tilde{Q}^{\dot{g}}$, $\tilde{M}^{i\dot{g}} = \hat{Q}^i\tilde{Q}^{\dot{g}}$, $M_1^{f\dot{g}} = Q^f\tilde{X}X\tilde{Q}^{\dot{g}}$, $P^{fg} = Q^f\tilde{X}Q^g$ and $\tilde{P}^{\dot{f}\dot{g}} = \tilde{Q}^{\dot{f}}X\tilde{Q}^{\dot{g}}$. The meson P is symmetric in flavor indices and $\tilde{P}$ is antisymmetric.

Note the matter content of this theory is similar to one of the models considered in [18], where $SU(N_c)$ gauge theory with matter corresponding to a symmetric and an antisymmetric multiplet, and m_f+8 fundamentals with $SU(m_f+8)_L$ flavor symmetry and m_f anti-fundamentals with $SU(m_f)_R$ flavor symmetry. Our model may be obtained by adding the $\hat{Q}$ dependent term of the superpotential (3.1) to that of [18]. Likewise, the model of [18], may be obtained from our model by adding mass terms of the form $\hat{Q}\tilde{Q}$, and integrating out the massive fermions. This deformation will be considered in more detail later in this section.

This theory has a non-anomalous $\mathbb{Z}_{8m_f}$ discrete symmetry generated by:

$$\begin{array}{rcl} (X,\ \tilde{X}) & \to & \alpha^{2m_f}\ (X,\ \tilde{X}) \\ \hat{Q} & \to & \alpha^{-m_f}\hat{Q} \\ (Q,\ \tilde{Q}) & \to & \alpha^{4-2N_c}(Q,\ \tilde{Q})\ , \end{array} \tag{3.3}$$

where $\alpha = e^{2\pi i/(8m_f)}$.

3.2 Magnetic theory

The theory has gauge group $SU(\tilde{N}_c)$ where $\tilde{N}_c = 3m_f+4-N_c$ and a symmetric and an antisymmetric chiral multiplet together with m_f+8 fundamental and m_f anti-fundamental chiral multiplets, and gauge singlet fields $M_0^{f\dot{g}}$, $\tilde{M}^{i\dot{g}}$, $M_1^{f\dot{g}}$, P^{fg} and $\tilde{P}^{\dot{f}\dot{g}}$. The fields transform under the symmetries as listed in table 2.

The superpotential takes the form:

$$W = \text{Tr }(Y\tilde{Y})^2 + \hat{q}\tilde{Y}\hat{q} + M_1q\tilde{q} + M_0q(\tilde{Y}Y)\tilde{q} + \tilde{M}\hat{q}\tilde{q} + Pq\tilde{Y}q + \tilde{P}\tilde{q}Y\tilde{q}\ . \tag{3.4}$$

The $\mathbb{Z}_{8m_f}$ discrete symmetry on the magnetic fields is generated by:

$$\begin{array}{rcl} (Y,\ \tilde{Y}) & \to & e^{i\pi Bm_f}\alpha^{2m_f}\ (Y,\ \tilde{Y}) \\ \hat{q} & \to & e^{i\pi Bm_f}\alpha^{-m_f}\hat{q} \\ (q,\ \tilde{q}) & \to & e^{i\pi Bm_f}\alpha^{4-2\tilde{N}_c}(q,\ \tilde{q})\ , \end{array} \tag{3.5}$$

where B is the baryon number operator. This symmetry is non-anomalous in the magnetic theory.

	$SU(\tilde{N}_c)$	$SU(m_f)_L$	$SU(m_f)_R$	$SO(8)_L$	$U(1)_B$	$U(1)_X$	$U(1)_R$
$\hat{q}$	$\tilde{\mathbf{N}}_\mathbf{c}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8_v}$	$\frac{1}{\tilde{N}_c}$	$\frac{N_c-m_f}{2\tilde{N}_c}$	$\frac{3}{4}$
q	$\tilde{\mathbf{N}}_\mathbf{c}$	$\overline{\mathbf{m_f}}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{1}{\tilde{N}_c}$	$\frac{m_f+2}{\tilde{N}_c}$	$1-\frac{\tilde{N}_c-2}{2m_f}$
$\tilde{q}$	$\overline{\tilde{\mathbf{N}}_\mathbf{c}}$	$\mathbf{1}$	$\overline{\mathbf{m_f}}$	$\mathbf{1}$	$-\frac{1}{\tilde{N}_c}$	$-\frac{m_f+2}{\tilde{N}_c}$	$1-\frac{\tilde{N}_c-2}{2m_f}$
Y	**asym**	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{\tilde{N}_c}$	$\frac{N_c-m_f}{\tilde{N}_c}$	$\frac{1}{2}$
$\tilde{Y}$	$\overline{\textbf{sym}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{\tilde{N}_c}$	$-\frac{N_c-m_f}{\tilde{N}_c}$	$\frac{1}{2}$
M_0	$\mathbf{1}$	$\mathbf{m_f}$	$\mathbf{m_f}$	$\mathbf{1}$	0	0	$2-\frac{N_c-2}{m_f}$
$\tilde{M}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{m_f}$	$\mathbf{8_v}$	0	$\frac{1}{2}$	$\frac{7}{4}-\frac{N_c-2}{2m_f}$
M_1	$\mathbf{1}$	$\mathbf{m_f}$	$\mathbf{m_f}$	$\mathbf{1}$	0	0	$3-\frac{N_c-2}{m_f}$
P	$\mathbf{1}$	**sym**	$\mathbf{1}$	$\mathbf{1}$	0	-1	$\frac{5}{2}-\frac{N_c-2}{m_f}$
$\tilde{P}$	$\mathbf{1}$	$\mathbf{1}$	**asym**	$\mathbf{1}$	0	1	$\frac{5}{2}-\frac{N_c-2}{m_f}$

Table 2: Magnetic theory: representation of fields.

3.3 't Hooft anomaly matching

As a check on the duality predicted by the brane configuration, we have checked the 't Hooft anomaly matching conditions and found that they are satisfied. The results are summarized in the following:

$$
\begin{array}{ll}
U(1)_R & -\frac{N_c^2}{2}-1 \\
U(1)_X & 3N_c \\
U(1)_B & 6 \\
U(1)_R^3 & \frac{1}{8}(7N_c^2-N_c-8)-\frac{1}{4m_f^2}(N_c-2)^3N_c \\
U(1)_X^3 & 0 \\
U(1)_B^3 & 0 \\
U(1)_X U(1)_B U(1)_R & -N_c-1 \\
U(1)_B^2 U(1)_R & -3 \\
U(1)_B^2 U(1)_X & 0 \\
U(1)_X^2 U(1)_B & 0 \\
U(1)_X^2 U(1)_R & -\frac{N_c(N_c+1)}{2} \\
U(1)_R^2 U(1)_B & 4 \\
U(1)_R^2 U(1)_X & 0 \\
SU(m_f)^3 & N_c d_3(m_f) \\
SO(8)^3 & 0 \\
SU(m_f)^2 U(1)_R & -\frac{N_c(N_c-2)}{4m_f} \\
SU(m_f)^2 U(1)_X & 0 \\
SU(m_f)^2 U(1)_B & \frac{1}{2} \\
SO(8)^2 U(1)_R & -\frac{N_c}{4} \\
SO(8)^2 U(1)_X & \frac{N_c}{2} \\
SO(8)^2 U(1)_B & 1
\end{array}
\tag{3.6}
$$

3.4 Superpotential deformations

Let us now consider deformations of the superpotential of the electric theory and examine how these are mapped to the magnetic theory. It is a further check on the duality that the resulting low-energy theories also form a dual pair.

If we deform the superpotential of the electric theory by a term of the form $mQ^1\tilde{Q}^1$, the low-energy theory takes the same form with $m_f \to m_f - 1$. In the magnetic theory, a term $mM_0^{1,1}$ is added to the superpotential. Taking the vevs of the mesons, $\hat{q}$, and q^g, $\tilde{q}^{\dot{g}}$ (for $g, \dot{g} > 1$) to vanish, the F-term equations reduce to

$$
\begin{aligned}
\tilde{q}^1 Y \tilde{Y} q^1 &= -m \\
\tilde{q}^1 q^1 &= 0 \\
q^1 \tilde{Y} q^1 &= 0 \\
\tilde{q}^1 Y \tilde{q}^1 &= 0 \\
Y \tilde{Y} Y &= 0 \\
\tilde{Y} Y \tilde{Y} &= 0\,,
\end{aligned}
\tag{3.7}
$$

and the D-terms take the form (2.2). A solution of these equations can be written in the form $Y_{12} = 1$, $Y_{21} = -1$, $\tilde{Y}_{23} = 1$, $\tilde{Y}_{32} = 1$, $(q^1)_3 = -\sqrt{2}$, $(\tilde{q}^1)_1 = \sqrt{2}$ and $\nu = 0$, with other entries zero. Here we have scaled m to 2 for convenience. The rank of the gauge group is reduced by 3, to $SU(3m_f - N_c + 1)$. The massless matter consists of a symmetric tensor, an antisymmetric tensor, 8 fundamental chiral multiplets, and $m_f - 1$ flavors of fundamental and anti-fundamental chiral multiplets. The low-energy magnetic theory is thus the dual of the low-energy electric theory.

The other deformation we consider is adding a mass term of the form $m\hat{Q}^1\tilde{Q}^1$ to the electric superpotential. On the magnetic side this corresponds to adding $m\tilde{M}^{1,1}$ to the superpotential. Taking the vevs of the mesons, $\hat{q}^i$ (for $i > 1$), q^g, $\tilde{q}^{\dot{g}}$ (for $g, \dot{g} > 1$) to vanish, the F-term equations now become

$$
\begin{aligned}
\tilde{q}^1 Y \tilde{q}^1 &= 0 \\
\tilde{Y} \hat{q}^1 &= 0 \\
\tilde{q}^1 \hat{q}^1 &= -m \\
\hat{q}^1_a \hat{q}^{1b} + 2(Y\tilde{Y}Y)_a^b &= 0 \\
\tilde{Y} Y \tilde{Y} &= 0\,.
\end{aligned}
\tag{3.8}
$$

Scaling m to $2\sqrt{2}$, a solution to these equations takes the form $(\tilde{q}^1)_1 = -2$, $(\hat{q}^1)_1 = \sqrt{2}$, $Y_{12} = 1$, $Y_{21} = -1$, $\tilde{Y}_{22} = 1$ with other components zero. This solves the D-term equations. The rank of the gauge group is broken by 2 to $SU(3m_f - N_c + 2)$. The massless matter consists of a symmetric tensor, an antisymmetric tensor, $m_f + 7$ fundamental chiral multiplets and $m_f - 1$ anti-fundamental chiral multiplets.

Giving mass to all 8 $\hat{Q}$'s in the electric theory leaves one with just the $\mathrm{Tr}\,(X\tilde{X})^2$ term in the superpotential. This is precisely one of the theories considered in [18]. On the magnetic side we see the gauge group is reduced to $SU(3m_f - N_c - 12)$ with m_f fundamentals and $m_f - 8$ anti-fundamentals, plus symmetric and antisymmetric tensors, which is precisely the magnetic dual found in [18]. We see therefore that our theory is related to that of [18] by a deformation. Likewise one could add the $\hat{Q}\tilde{X}\hat{Q}$ term to the superpotential of [18], to recover the theory considered here.

3.5 Flat directions

In this subsection we consider the low-energy theories on the baryonic branches of our original theory. The electric and magnetic descriptions of these branches will be shown to form dual pairs, providing a further check on the duality of the original theory.

Consider the branch where the baryon $\tilde{B}_n = \tilde{X}^n \tilde{Q}^{N_c-n} \tilde{Q}^{N_c-n}$ gets an expectation value. Here the color indices are contracted with two ϵ-tensors. In the magnetic theory this is mapped to $\tilde{b}_n = \tilde{Y}^{2m_f+4-n} \tilde{q}^{m_f-N_c+n} \tilde{q}^{m_f-N_c+n}$. This expectation value breaks the gauge group to $SO(n)$ on the electric side and leaves us with $2m_f$ massless fundamentals (the superpotential generates a mass term for the adjoint matter field which may be integrated out). In addition, there are a number of singlets that decouple. On the magnetic side we likewise have the gauge group broken to $SO(2m_f + 4 - n)$ with $2m_f$ fundamentals. This is the magnetic dual of the electric theory as expected [8].

From the brane point of view, the baryonic branch with $n = N_c$ corresponds to moving the central NS-fivebrane in the positive x^7 direction. The displacement of the central fivebrane in the x^7 direction should be identified with the would-be FI parameter $-\nu$ appearing in the D-term equations (2.2). From this equation it is clear that X ($\tilde{X}$) can get an expectation value for ν positive (negative). A positive displacement generates a mass for the fundamentals coming from the half D-sixbranes and the D-fourbranes intersect the orientifold sixplane on the side where the gauge group is projected down to $SO(N_c)$. However, the brane picture only gives one real parameter for this baryonic branch. Presumably if this picture is lifted to M-theory, a more careful analysis of the curve along the lines of [19] will give a complete picture of the moduli space.

Now consider the baryonic branch where $B_n = X^n Q^{N_c-2n}$ gets an expectation value. This breaks the gauge group to $Sp(n)$ and leaves us with $2m_f + 8$ massless fundamentals, plus some singlets that decouple. The superpotential takes the form $W = \mathrm{Tr}\ \hat{Q}^4/\lambda$, where λ is proportional to the vev of X. On the magnetic side the baryon operator is mapped to $b_n = Y^{m_f+2-n} q^{m_f-N_c+2n}$. This breaks the gauge group to $Sp(m_f + 2 - n)$ and leaves us with $2m_f + 8$ massless fundamentals plus some singlets. This theory is the magnetic dual expected from [8], with an additional deformation of the superpotential, which vanishes in the limit $\lambda \to \infty$.

In terms of branes, for $n = N_c/2$ (with N_c even), this corresponds to moving the central NS-fivebrane in the negative x^7 direction. The 8 special fundamentals remain massless on this branch, and the gauge group is projected to $Sp(N_c/2)$ when the D-fourbranes intersect this section of the O-sixplane. Again we only see one real parameter of this branch in a simple way in the brane picture.

Finally, let us consider the baryonic branches where $\hat{B}_{n,a} = X^n \hat{Q}^a Q^{N_c-2n-a}$ gets an expectation value. The gauge group is broken to $Sp(n)$ as before, but the massless matter content is different in this case. The F-terms give

$$
\begin{array}{rcl}
\tilde{X}\hat{Q} & = & 0 \\
\tilde{X}X\tilde{X} & = & 0 \\
\hat{Q}_a\hat{Q}^b + 2(X\tilde{X}X)_a^b & = & 0 \; ,
\end{array}
\tag{3.9}
$$

which implies that both X and $\tilde{X}$ receive a vev. Solving these equations we find the massless matter is $2m_f + 8 - 2a$ fundamentals of $Sp(n)$. The baryon is mapped to $\hat{b}_{n,a} = Y^{m_f+2-n-a}\hat{q}^a q^{m_f-N_c+2n+a}$ on the magnetic side. Likewise, this breaks the gauge group to $Sp(m_f + 2 - n - a)$ with $2m_f + 8 - 2a$ massless fundamentals. This is the expected magnetic dual [8].

4 Conclusions and outlook

We investigated a brane configuration that genuinely gives chiral matter in its effective four-dimensional field theory description. The key ingredient is the orientifold sixplane that is divided in two by an NS-fivebrane. Using linking number arguments along the lines of [9] we were able to obtain the dual model from brane moves. This is of course only a first step towards gaining a deeper understanding of chiral $\mathcal{N} = 1$ theories from branes.

Still in our model there remain many unanswered questions. We considered the brane configuration only in Type IIA string theory. In order to obtain more information about the strong coupling behavior one should lift the configuration to M-theory and derive a complex curve describing the system. The existence of a smooth curve seems plausible since the sixbrane charge does not jump at the location of the center NS-fivebrane thanks to the additional eight fundamentals $\hat{Q}$. The role these special fields play is very interesting. From the field theory point of view there is actually no reason why one should couple precisely eight of the fundamentals to the symmetric tensor. Indeed we saw that our model can also be seen as a deformation of the model in [18] and vice versa. A brane configuration giving rise to a coupling of less (or more) than eight fundamentals to the symmetric tensor needs the introduction of additional D-eightbranes. This is because the sixbrane charge would jump now at the location of the center NS-fivebrane and this is only consistent in the presence of D-eightbranes [10].

Another generalization is to consider also configurations with coinciding fivebranes at the positions of our A-, B- and C-fivebranes. These should give field theories with superpotentials of the form Tr $(X\tilde{X})^{2(k+1)}$. Putting additional D-sixbranes on top of the orientifold presumably gives theories with couplings not only of fundamentals to the symmetric tensor but also of anti-fundamentals to the antisymmetric tensor. All these points deserve further investigation and we plan to elaborate on these issues in future work [20].

Acknowledgments

We wish to thank C. Gomez, A. Hanany, K. Intriligator and E. Rabinovici for discussions. We thank C.E.R.N. and I.F.T. at U.A.M. for hospitality during the completion of this work. K.L. was supported by FWF Project Nr. P10268-PHY. E.L. was supported by Lise Meitner Fellowship M456-TPH. D.L. was supported in part by DOE grant DE-FG0291ER40688-Task A.

References

[1] J. Polchinski, *Dirichlet Branes and Ramond-Ramond Charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [`hep-th/9510017`] and *TASI Lectures on D-branes,* preprint NSF-ITP-96-145 `hep-th/9611050`.

[2] A. Strominger, *Open p-Branes*, *Phys. Lett.* **B 383** (1996) 44; P. Townsend, *D-Branes from M-Branes*, *Phys. Lett.* **B 373** (1996) 68 [`hep-th/9512062`].

[3] E. Witten, *Solutions of four-dimensional Field Theories via M-Theory*, *Nucl. Phys.* **B 500** (1997) 3 [`hep-th/9703166`].

[4] A. Hanany and E. Witten, *Type IIB Superstrings, BPS Monopoles, and Three-Dimensional Gauge Dynamics*, *Nucl. Phys.* **B 492** (1997) 152 [`hep-th/9611230`].

[5] K. Landsteiner and E. Lopez, *New Curves from Branes,* `hep-th/9708118`.

[6] J.L.F. Barbon, *Rotated Branes and* $\mathcal{N} = 1$ *Duality*, *Phys. Lett.* **B 402** (1997) 59 [`hep-th/9703051`].

[7] S. Elitzur, A. Giveon and D. Kutasov, *Branes and* $\mathcal{N} = 1$ *Duality in String Theory*, *Phys. Lett.* **B 400** (1997) 269 [`hep-th/9702014`];
S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and A. Schwimmer, *Brane Dynamics and* $\mathcal{N} = 1$ *Supersymmetric Gauge Theory*, *Nucl. Phys.* **B 505** (1997) 202 [`hep-th/9704104`].

[8] N. Seiberg, *Electric-Magnetic Duality in Supersymmetric Non-Abelian Gauge Theories*, *Nucl. Phys.* **B 435** (1995) 129 [`hep-th/9411149`].

[9] J.H. Brodie and A. Hanany, *Type IIA Superstrings, Chiral Symmetry and* $\mathcal{N} = 1$ *4D Gauge Theory Dualities*, *Nucl. Phys.* **B 506** (1997) 157 [`hep-th/9704043`].

[10] A. Hanany and A. Zaffaroni, *Chiral Symmetry from Type IIA Branes*, *Nucl. Phys.* **B 509** (1998) 145 [`hep-th/9706047`].

[11] J. Lykken, E. Poppitz and S. P. Trivedi, *Chiral Gauge Theories from D-Branes,* `hep-th/9708134`; *M(ore) on Chiral Gauge Theories from D-Branes,* `hep-th/9712193`.

[12] N. Evans, C.V. Johnson and A. Shapere, *Orientifolds, Branes and Duality of 4D Gauge Theories,* *Nucl. Phys.* **B 505** (1997) 251 [hep-th/9703210].

[13] K. Landsteiner, E. Lopez and D.A. Lowe, $\mathcal{N}=2$ *Supersymmetric Gauge Theories, Branes and Orientifolds,* *Nucl. Phys.* **B 507** (1997) 197 [hep-th/9705199].

[14] A. Brandhuber, J. Sonnenschein, S. Theisen and S. Yankielowicz, *M Theory and Seiberg-Witten Curves: Orthogonal and Symplectic Groups,* *Nucl. Phys.* **B 504** (1997) 175 [hep-th/9705232];
C. Ahn, K. Oh and R. Tatar, *Branes, Geometry and* $\mathcal{N}=1$ *Duality with Product Gauge Groups of SO and Sp,* hep-th/9707027;
C. Csaki and W. Skiba, *Duality in Sp and SO Gauge Groups from M Theory,* *Phys. Lett.* **B 415** (1997) 31 [hep-th/9708082]; *$Sp(N_c)$ Gauge Theories and M Theory Fivebrane,* hep-th/9708127; *M Theory Fivebrane Interpretation for Strong Coupling Dynamics of $SO(N_c)$ Gauge Theories,* *Phys. Lett.* **B 416** (1998) 75 [hep-th/9709096]; *M Theory Fivebrane and Confining Phase of* $\mathcal{N}=1$ *$SO(N_c)$ Gauge Theories,* hep-th/9712005;
C. Ahn, *Confining Phase of* $\mathcal{N}=1$ *$Sp(N_c)$ Gauge Theories via M Theory Fivebrane,* hep-th/9712149;
S. Terashima, *Supersymmetric Gauge Theories with Classical Groups via M Theory Fivebrane,* hep-th/9712172.

[15] E. Witten, *Toroidal Compactification without Vector Structure,* hep-th/9712028.

[16] N. Seiberg, *IR Dynamics on Branes and Space-Time Geometry,* *Phys. Lett.* **B 384** (1996) 81 [hep-th/9606017].

[17] N. Seiberg and E. Witten, *Gauge Dynamics and Compactification to Three Dimensions,* hep-th/9607163.

[18] K. Intriligator, R.G. Leigh and M.J. Strassler, *New Examples of Duality in Chiral and Non-Chiral Supersymmetric Gauge Theories,* *Nucl. Phys.* **B 456** (1995) 567 [hep-th/9506148].

[19] K. Hori, H. Ooguri and Y. Oz, *Strong Coupling Dynamics of Four-Dimensional* $\mathcal{N}=1$ *Gauge Theories from M Theory Fivebrane,* hep-th/9706082.

[20] K. Landsteiner, E. Lopez and D.A. Lowe, work in progress.

JHEP02(1998)007